

To Solve Polynomial Inequalities

1. Move everything to one side and solve the equality (Factor, complete the square, quadratic formula if you must)
2. Put zeros on number line
3. Test regions/Apply Understanding of Bouncing + Crossing + Polynomial End Behavior
4. Check answer

Unit 1 Day 6 (1.5)

- Polynomial Inequalities
- Domain

Homework: Polynomial Inequalities

Quiz Next Class:

See Canvas for a list of topics

$$(x-3)^2(x+1) > 0$$

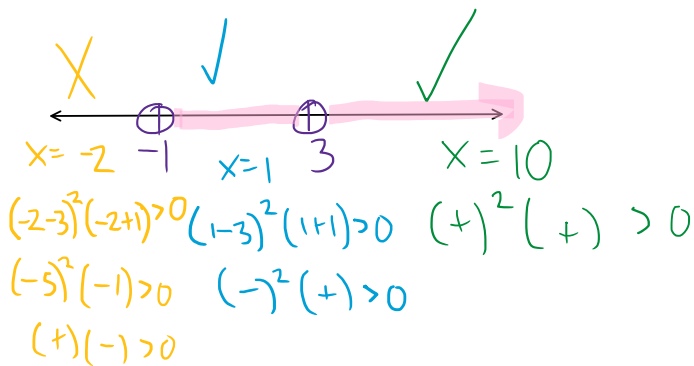
$$(x^2-2x-3)(x-3) = 0$$

$$(x-3)(x+1)(x-3) = 0$$

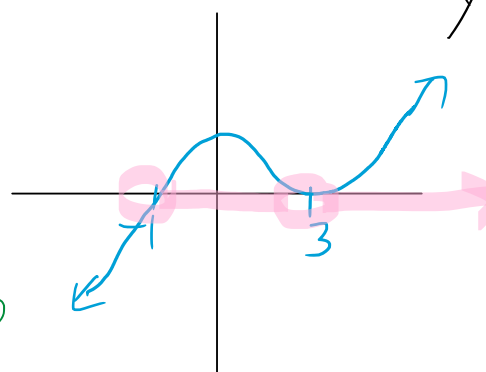
$$(x-3)^2(x+1) = 0$$

Solving By Testing Regions

Solving By Graphing



$$(-1, 3) \cup (3, \infty)$$



$$y = (x^2-2x-3)(x-3)$$

$$y = (x-3)^2(x+1)$$

Examples - Solve each of the following inequalities. Express your answer in interval notation.

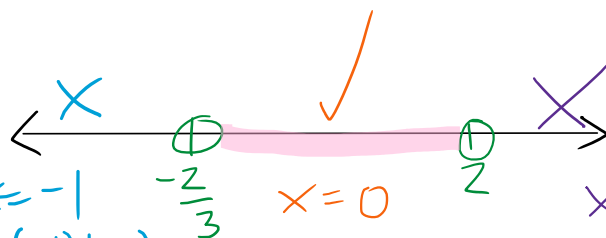
A) $4x + 4 > 3x^2$

$$0 > 3x^2 - 4x - 4$$

$$0 > (3x + 2)(x - 2)$$

$$x = -\frac{2}{3}$$

$$x = 2$$



$$x = -1$$

$$0 > (-1)(-3)$$

$$0 > (-)(-)$$

$$0 > +$$

$$x = 0$$

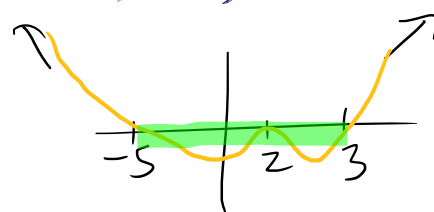
$$0 > (2)(-2)$$

$$x = 3$$

$$0 > (11)(1)$$

$$0 > 11$$

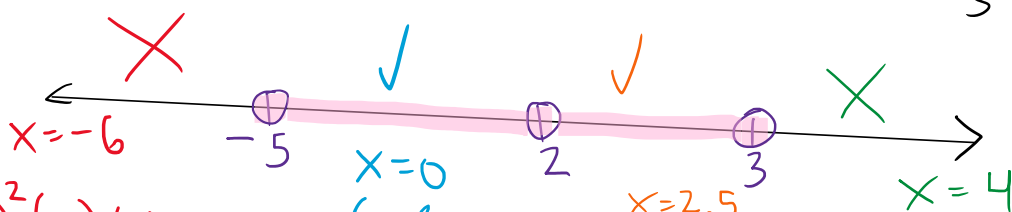
$$\left(-\frac{2}{3}, 2\right)$$



B) $(x^2 - 5x + 6)(x^2 + 3x - 10) \leq 0$

$$(x - 2)(x - 3)(x + 5)(x - 2) \leq 0$$

$$(x - 2)^2(x - 3)(x + 5) \leq 0$$



$$(-)^2(-)(-) \leq 0$$

$$(+)(-)(-) \leq 0$$

$$(-)^2(-)(+) \leq 0$$

$$(+)(-)(+) \leq 0$$

$$x = 2.5$$

$$(+)^2(-)(+) \leq 0$$

$$(+)(-)(+) \leq 0$$

$$(+)^2(+)(+) \leq 0$$

$$[-5, 3]$$

Examples - Solve each of the following inequalities. Express your answer in interval notation.

C) $2x^4 - 5x^2 < 3x^3$

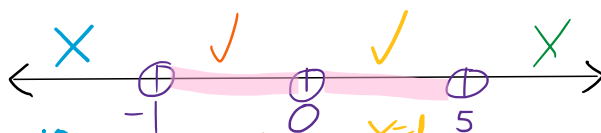
$$2x^4 - 3x^3 - 5x^2 < 0$$

$$(x)^2(2x^2 - 3x - 5) < 0$$

$$(x)^2(2x - 5)(x + 1) < 0$$

D) $(x^2 - 4x + 5)(x^2 - 5x + 4) \geq 0$

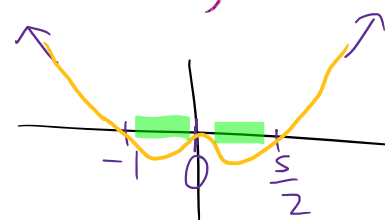
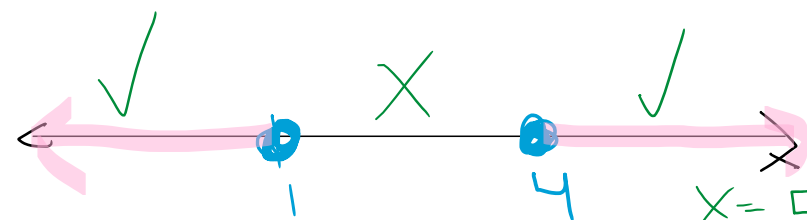
$$(x^2 - 4x + 5)(x - 4)(x - 1) \geq 0$$



$x = -10$
 $(-)^2(-)(-) < 0$
 $(+)(-)(-) < 0$

$x = -\frac{1}{2}$
 $(-)^2(-)(+) < 0$
 $(+)(-)(+) < 0$

$x = 1$
 $(+)^2(-)(+) < 0$
 $(+)^2(+)(+) < 0$



$(-\infty, 1] \cup [4, \infty)$

$(+)(+)(+) \geq 0$

Examples - Solve each of the following inequalities. Express your answer in interval notation.

$$\text{E) } (2 - x)^3(3x - 1)^2(x + 4)(x + 1)^2 \geq 0$$

$$\text{F) } 2x^4 + 13x^2 + 20 < 0$$

Determine the domain of each of the following

$$f(x) = \sqrt{x} \quad \text{Domain:} \quad x \geq 0$$

$$g(x) = \sqrt{x - 4}$$

$$\text{Domain:} \quad \begin{aligned} x - 4 &\geq 0 \\ \rightarrow x &\geq 4 \end{aligned}$$

$$h(x) = \sqrt{3 - x}$$

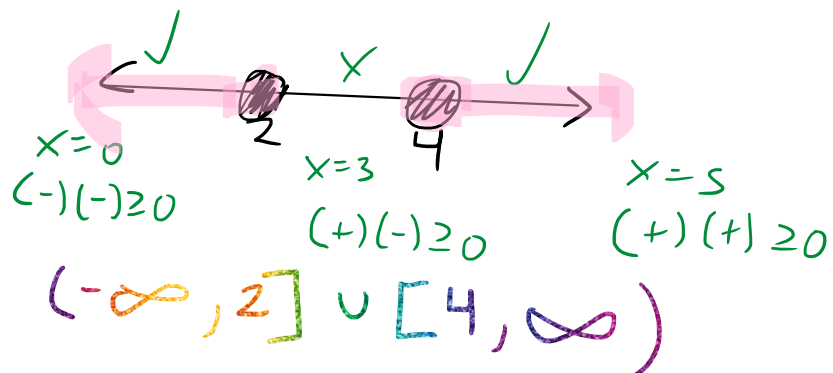
$$\begin{aligned} \text{Domain:} \quad 3 - x &\geq 0 \\ -x &\geq -3 \\ x &\leq 3 \end{aligned}$$

Determine the domain of each of the following

$$f(x) = \sqrt{x^2 - 6x + 8}$$

$$x^2 - 6x + 8 \geq 0$$

$$(x-2)(x-4) \geq 0$$

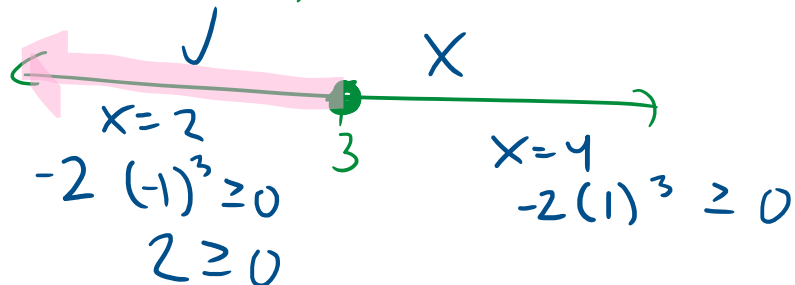


$$g(x) = \sqrt{(-2x + 6)(x^2 - 6x + 9)}$$

$$-2(x-3)(x-3)(x-3) \geq 0$$

$$(-\infty, 3]$$

$$-2(x-3)^3 \geq 0$$



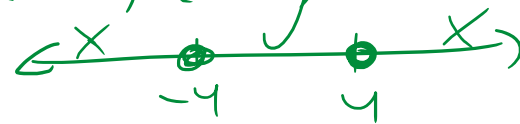
More Domain Practice - Determine the domain of each of the following

$$f(x) = \sqrt{16 - x^2}$$

$$16 - x^2 \geq 0$$

$$0 \geq x^2 - 16$$

$$0 \geq (x-4)(x+4)$$



$$g(x) = \sqrt{(1-2x)^3(4-x)(16-x^2)(1+x)}$$

$$h(x) = \sqrt[3]{2x^4 - 13x^2 - 45}$$

Domain: $(-\infty, \infty)$

More Domain Practice - Determine the domain of each of the following

$$f(x) = \sqrt[4]{2x^4 - 13x^2 - 45}$$

$$g(x) = \sqrt[3]{(x^2 - 4x + 4)(x^2 - 4)}$$

$$h(x) = \sqrt{(-x^2 - 9)(x + 3)}$$

Sketching

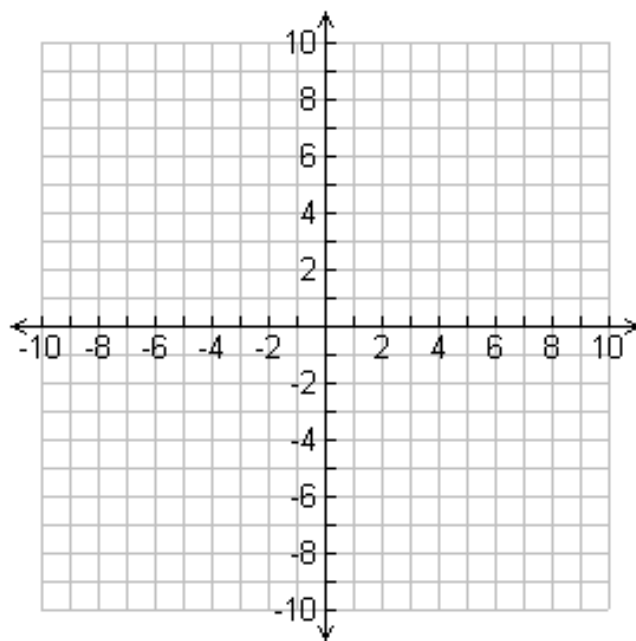
Sketch a continuous function $f(x)$ with the following properties:

- $f(x) \geq 0$ on the interval $(-\infty, -1)$
- $f(x)$ is a cubic polynomial
- $f(0) = -4$ is a local minimum
- The rate of change of $f(x)$ is decreasing on $(1, \infty)$

Is it possible $f(x)$ is an odd function?

Is it possible $f(x)$ has two nonreal zeros?

Is it possible $f(x)$ has no nonreal zeros?

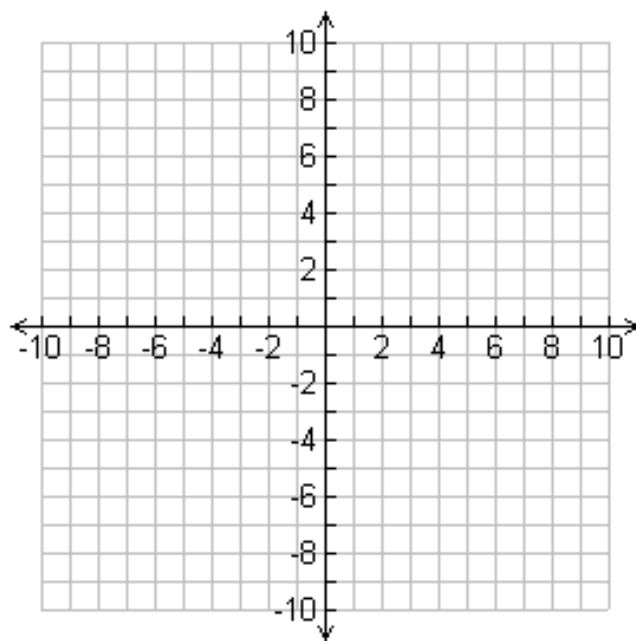


Sketching

Sketch a continuous function $g(x)$ with the following properties:

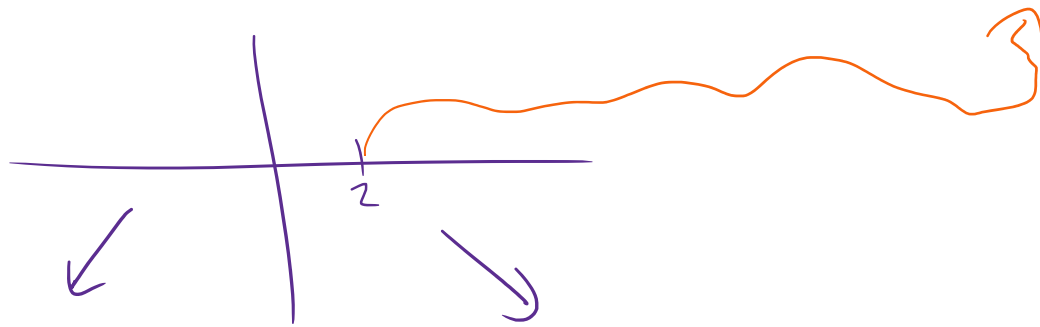
- On the interval $(3, \infty)$, $g(x) < 0$
- $g(x)$ is an even function
- The global maximum of $g(x)$ is 2
- On the interval $(0,1)$, $g(x)$ is increasing and concave up.

Describe the end behavior of $g(x)$ using limit notation.



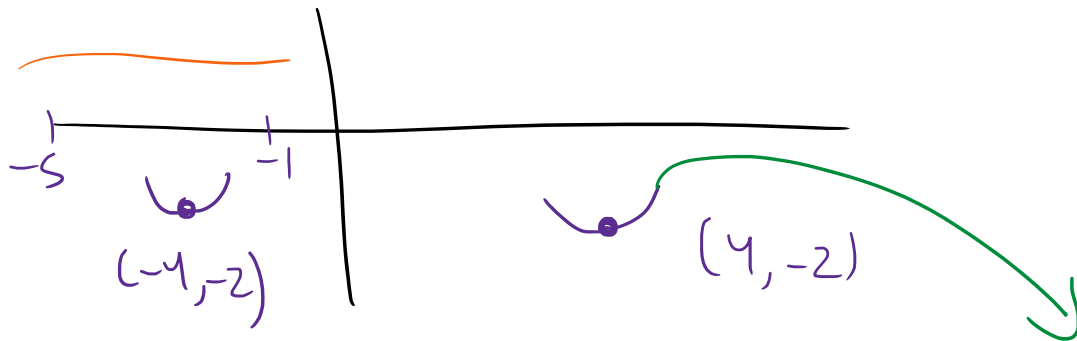
Example 9a: Below I tried to give four attributes of a polynomial function. Turns out, by putting these together, I've described something impossible. Why is the scenario impossible? It may help to try to sketch the scenario.

- * • $\lim_{x \rightarrow -\infty} f(x) = -\infty$
- * • f is a 4th degree polynomial
- On the interval $(2, \infty)$, $f(x) \geq 0$
- On the interval $(0, 2)$, $f(x)$ is increasing with an increasing rate of change.



Example 9b: Below I tried to give four attributes of a polynomial function. Turns out, by putting these together, I've described something impossible. Why is the scenario impossible? It may help to try to sketch the scenario.

- $\lim_{x \rightarrow \infty} g(x) = -\infty$
- $g(x)$ is an even function
- On the interval $(-5, -1)$, $g(x) > 0$
- $g(x)$ has a relative minimum at the point $(4, -2)$



Example 9c: Below I tried to give four attributes of a polynomial function. Turns out, by putting these together, I've described something impossible. Why is the scenario impossible? It may help to try to sketch the scenario.

- $f(x)$ is an odd function
- On the interval $(-3, 1)$, $f(x) < 0$
- * As the input of f increases without bound, the output of f decreases without bound
- f has no local extrema

$$f\left(\frac{1}{2}\right)$$

$$f\left(-\frac{1}{2}\right)$$

